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Analytical determination and numerical computation of extinction coefficients for vegetation with given leaf distribution

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ABSTRACT

A study has been carried out, with both analytical and numerical methods, on the effective extinction coefficient of a medium featuring a given vegetation. The studied domain contains a heterogeneous medium involving in successive cases: leaves under planophile, erectophile, plagiophile, extremophile orientations, and a mixing of these conditions, all situations assuming a uniform distribution in space. A ray tracing method has been used in order to identify an effective extinction coefficient after computation of the mean free path. For these academical situations, comparisons with some analytical formulations have been carried out in order to achieve the validation of the numerical method. Then, the role of a heterogeneous distribution in space has been studied, introducing morphological properties (orientation and spatial heterogeneities) which produce a set of leaves featuring a sort of virtual vegetation, numerically built in a box. In particular, conditions of leaves mainly located at the periphery of a crown have been considered. Numerical results show that analytical relations for the extinction coefficient may be used for the different leaf orientations, when the spatial distribution is assumed homogeneous. However, some discrepancies from these solutions appear when heterogeneous distributions in space are used, suggesting that such relationships will become less accurate when applied on a true vegetation. The numerical tool is now ready and will be used on a vegetation built with true morphological characteristics.

Introduction

When modeling forest fire propagation, one peculiar difficulty is related to the radiative transfer role. The fact that radiation is a major propagation actor is well known, but the way it has to be taken into account is an open research question. It is obvious that a complete fire propagation model has to involve not only radiative transfer, but also numerous combined sub-problems for heat and mass transport, combustion phenomena, turbulence, etc., with supplementary difficulties brought by the heterogeneous structure of the medium under study and the multi scale situation of the problem (see for example [1] for a recent overview and contribution on that topic). Some progress related to each sub-model is then required before gathering all the dedicated tools in a combined simulation. In this frame we are studying radiative transfer in order to improve our understanding of the involved mechanisms and also in order to provide input data directly useful for fire models. Simulation of radiative transfer is often considered as a hard task alone, in particular due to the complexity of this heat transfer mode and to a lack of knowledge relative to the required input properties (radiative properties of the vegetation and spectral distribution of the radiation emitted by flames for the present domain). The non-gray nature of the radiative transfer is obvious, although often omitted for the sake of computational cost reduction. In a recent contribution present authors have presented a preliminary study of the radiative properties of the vegetation, in which experimentations have been carried out on six species of the Mediterranean region [2]. Spectral transmissivity and reflectivity have been measured and the absorptivity of a single leaf has been deduced. Then a ray tracing method has been used in order to identify an extinction coefficient of a medium built numerically with given leaves uniformly distributed in space and with random orientation. This coefficient can be considered as an effective extinction coefficient corresponding to an equivalent homogeneous medium. The sharp spectral variations of the radiative properties of the studied species have been clearly observed. Discrepancy from the currently used assumption of a vegetation behaving like a black radiative surface has been studied and comparisons with the well known De Mestre's relationship [3] have been performed. It has been shown that even when the vegetation properties shift from the blackbody ideal case, an easy correction of the relationship can be done multiplying the extinction coefficient by a factor equal to $1-\tau$, where τ is the transmissivity of the vegetation species.

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Nomenclature

l	mean free path before absorption event	m
$\mathcal{L}_{Tot}$	total path traveled by rays	m
n_l	leaf density	number of leaves · m^{-3}
N_l	number of leaves	
$nb_{extinct}$	total number of extinction events	m^{-3}
$p(\varphi,\theta)$	probability density of $P(\varphi,\theta)$	sr^{-1}
$P(\varphi,\theta)$	probability to be oriented according to φ and θ	
S	one-side area of a vegetation element	m^2
$\mathcal{S}_{obs}$	mean obstruction area of a leaf	m^2
S_t	total area of a vegetation element	m^2

Greek symbols

α	absorptivity	
β	extinction coefficient	m^{-1}
ρ	reflectivity	
τ	transmissivity	
θ	azimuthal angle measured from the x-axis	rad
φ	polar angle measured from the z-axis	rad

List of abbreviations

LAI	Leaf Area Index
RTE	Radiative Transfer Equation

This correction becomes important in some definite wavelength domains. However, when considering total coefficients, averaged over the whole spectral range, the deviation from the original relationship has been found to be moderate: the multiplying factor lying between 0.94 and 1. The limitation of this previous study was linked to an assumption of leaves distributed in a homogeneous manner in space.

In order to extend our analysis a numerical study has been carried out, no more only involving homogeneously distributed leaves, but focusing on the influence of the leaf distribution regarding the orientation or the spatial location of the leaves. This remains an analytical and numerical exercise, not yet performed on actual morphological properties of a true vegetation, but this is the last preliminary step before going on with numerically built vegetation species.

In the literature devoted to fire applications, the extinction coefficient is almost always simplified to its basic contribution predicted by the De Mestre's relationship. However, some contributions focused on radiative transfer through vegetation, taking into account the effect of an heterogeneous distribution of leaves, can be found in the domains of remote sensing or ecology for example. The early study by Monsi and Saeki [4] has been followed by numerous works focused on the interaction between solar irradiation and vegetation. Heterogeneous media can be for example simulated by multiple layers [5] allowing the prediction of sun radiation absorption. The problem of reflectance prediction by a canopy has also received a wide attention. Concerning the problems raised by the interactions between radiation and vegetation, the review by Myneni et al. [6] gives an overview on the theory of radiative transfer in canopies, including the role of scattering or the influence of the heterogeneity in the leaf distribution. In comparison with the domains of remote sensing or ecology some different problems are raised in the study of radiative transfer during fire propagation. First of all, the reflection or the absorption of irradiation is not sufficient for modeling the thermal source term due to radiative transfer. For the solution of the radiative transfer problem an extinction coefficient is sought, characterizing the medium equivalent to the vegetation, and the interaction with radiation has to be sought inside the vegetation itself. Secondly, when the fire source is close to the vegetation the irradiation is not collimated but may come in various directions in a diffuse manner, making the study of the role of orientation so important. A third particularity is the wavelength range, since flames are emitting in a large manner in the infrared, where it has been shown through measurements that the vegetation matter almost completely absorbs the radiation (absorptivity near constant and above 97% for wavenumbers under 4000 cm^{-1} and averaged absorptivity for various species typically between 90% and 95% [2]). This assumption of complete absorption would not be acceptable for a study in the visible for example, or at some definite wavelengths where scattering or reflection effects could not be omitted. For a study over a large range in the infrared, this can be considered as a sufficient approximation. These observations are guiding some particularities of the present study: (i) we will use a numerical tool that describes the radiation propagation inside a set of given leaves in order to identify an extinction coefficient; (ii) moreover, it has to potentially take account of orientation and spatial distribution; (iii) finally, purely absorbing leaves can be considered, since reflection and scattering will not be addressed owing to the wavelength range of interest.

In the present paper, the problem of leaf orientation will be first dealt with. Situations of particular vegetation types will be treated, involving planophile, plagiophile, erectophile, extremophile leaf distributions, and mixing of these situations. For such cases, analytical derivation of relationships yielding the extinction coefficient is possible. This will be presented and comparisons will be carried out between the corresponding results and numerical data obtained from a ray tracing process. This will provide a validation of our numerical identification method. Then the case of a heterogeneous distribution in space will be investigated, simulating leaves distributed in a crown, or on the periphery of the crown. With our previous spectral analysis and the present numerical step, the prediction tool will be ready for the simulation of actual trees, involving the spatial distribution of leaves through the use of true morphological characteristics.

The following sections will be devoted successively to the derivation of the analytical formulations when possible, to the presentation of the ray tracing method, and finally to the numerical results and the related analysis.

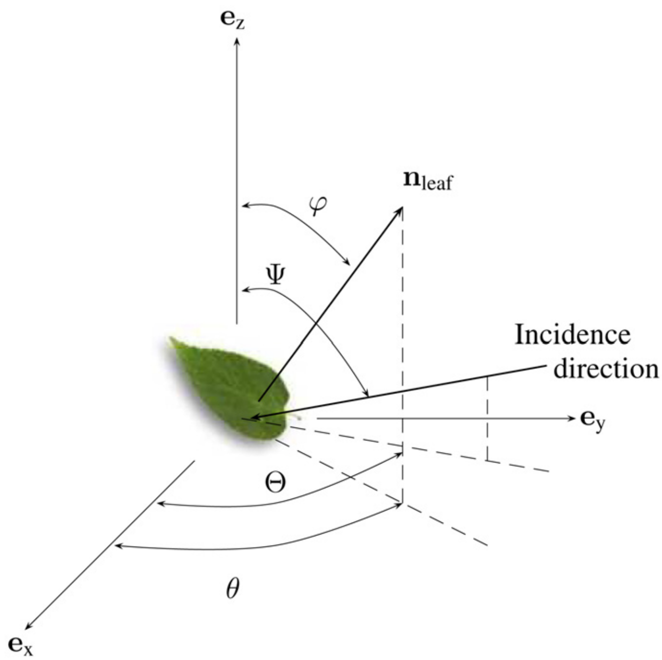
1. Models for leaf angle distribution functions

The leaf angle distribution function, $p(\varphi,\theta)$, is defined as the probability density of leaf orientation according to the polar (φ) or/and the azimuthal angle (θ). This function expressed by the fraction of leaf area per unit leaf solid angle around a given direction, is normalized as

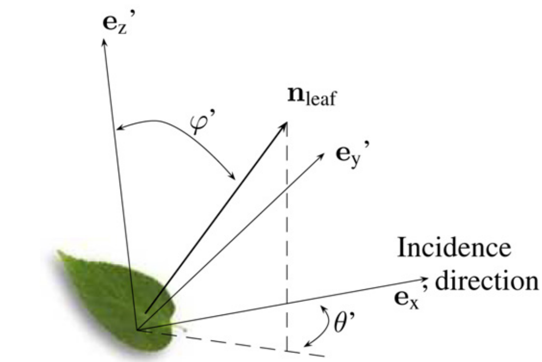
$$\frac{1}{4\pi}\int_{\varphi=0}^{\varphi=\pi}\int_{\theta=0}^{\theta=2\pi} p(\varphi,\theta)\sin\varphi\,\mathrm{d}\theta\,\mathrm{d}\varphi = 1 \tag{1}$$

The polar φ and azimuthal θ angles are shown on Fig. 1(a), related to the leaf normal vector ($\mathbf{n}_{leaf}$) and the coordinate system ($\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$).

Several models exist in order to represent accurately realistic orientation of vegetation media: De Wit's functions [7], Beta distribution function [8], ellipsoidal function [9], rotated-ellipsoidal function [10], Verhoef's algorithm [11]. These different probability densities are currently used when studying radiative transfer in vegetation canopies or in the frame of remote sensing.



(a) Representation of the polar and azimuthal angles



(b) Representations of the new coordinate system and the view angles

Fig. 1. Coordinate systems.

In the present study, the De Wit's functions [7] are used. Their expressions are simpler than the others and allow the analytical development of the extinction coefficient. Azimuthal symmetry being assumed, De Wit's suggestion [7] allows to classify the leaves according to five categories as a function of leaf orientation:

- Uniform – Each possible leaf angle has the same probability to occur

$$p_{\mathrm{Unif}}(\varphi) = 1 \tag{2}$$

- Planophile – The leaves are mainly horizontal

$$p_{\mathrm{Plano}}(\varphi) = \frac{3}{2}(1 + \cos 2\varphi) \tag{3}$$

- Erectophile – The leaves are mainly vertical

$$p_{\mathrm{Erect}}(\varphi) = \frac{3}{4}(1 - \cos 2\varphi) \tag{4}$$

- Plagiophile – The leaves are mainly inclined (with a 45-degree-orientation)

$$p_{\mathrm{Plagio}}(\varphi) = \frac{30}{32}(1 - \cos 4\varphi) \tag{5}$$

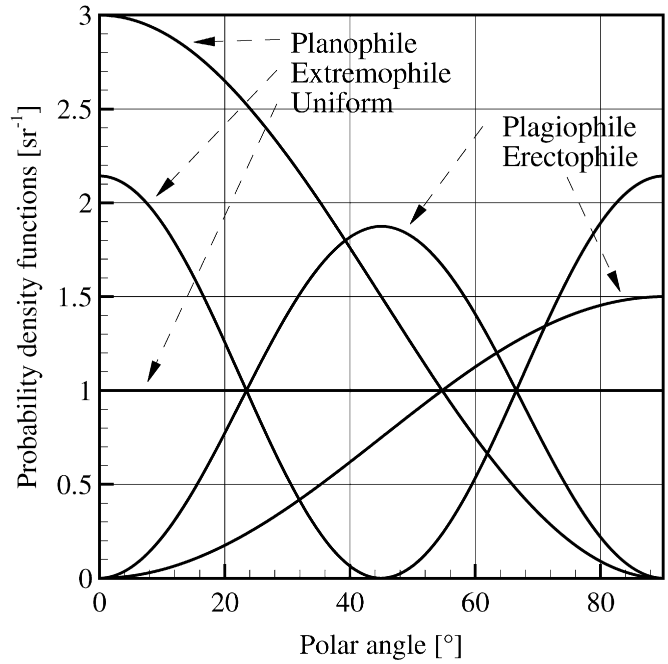


Fig. 2. Representation of the different probability densities.

- Extremophile – The leaf orientation appears according to two modes (mainly horizontal and vertical)

$$p_{\mathrm{Extre}}(\varphi) = \frac{30}{28}(1 + \cos 4\varphi) \tag{6}$$

These expressions may vary according to the angle definitions (choice of the polar or zenithal angles). On Fig. 2, the different probability densities are presented as a function of the polar angle in the range between 0 and 90° (for symmetry reasons).

The advantage of the De Wit's functions is that their expressions are simple and that they do not depend on any parameter, except the polar angle. However, the main drawback is that no realistic canopy exactly matches these representations. In recent works, Biswas [12] developed some new analytic leaf angle distribution models applying a linear combination of the De Wit's functions as

$$p_{\mathrm{real}}(\varphi) = a p_{\mathrm{Plano}}(\varphi) + b p_{\mathrm{Erect}}(\varphi) + c p_{\mathrm{Plagio}}(\varphi) \tag{7}$$

This relation needs three parameters (a, b, c) which can be adjusted according to the studied vegetation. The extremophile contribution is not directly included, because it can be taken into account with the planophile and erectophile contributions. Following the normalization condition (Eq. (1)), a, b and c must satisfy the relation

$$a + b + c = 1 \tag{8}$$

Regarding the radiative transfer, these works [12] demonstrate that a real situation can be actually described by this new relationship (Eq. (7)) in preserving a simple formalism.

2. Analytical study

This section is devoted to the analytical derivation of the extinction coefficient obtained for a given vegetation orientation. In a previous work [2] the analytical background of the De Mestre's relationship has been discussed for uniformly distributed and oriented leaves, with a demonstration based on mathematical considerations. In this case, the extinction coefficient only depends on the leaf density and the total surface of one leaf. However, as radiative transfer is a directional transfer, the extinction coefficient is also expected to be a function of the propagation direction. Indeed, the extinction coefficient is the same whatever the direction for randomly oriented leaves, but it becomes a function of the polar angle characterizing the propagation direction when the leaf orientation is given by an anisotropic probability density function (azimuthal symmetry being assumed). Then, the case of planophile, erectophile or other vegetation types can be addressed. This is aimed to provide a realistic and fine extinction coefficient, which could be introduced in the RTE (Radiative Transfer Equation) solution.

The method presented in [2] is followed for the given orientations presented through formulae (2)–(6). The mean projected area of one leaf $\mathcal{S}_{\mathrm{obs}}$ (according to $\mathbf{e}_{\mathrm{x}}$) is estimated in taking into account all possible orientations

$$\mathcal{S}_{\mathrm{obs}} = \frac{S}{4\pi} \int_{\varphi=0}^{\varphi=\pi} \int_{\theta=0}^{\theta=2\pi} p(\varphi,\theta) |\cos\theta| |\sin\varphi| \sin\varphi \, \mathrm{d}\theta \, \mathrm{d}\varphi \tag{9}$$

where S is the area of one side of the leaf.

The extinction coefficient can be considered as the sum of all leaf contributions and can be expressed as

$$\beta = n_l \mathcal{S}_{\mathrm{obs}} \tag{10}$$

When the calculation is done for a uniform orientation $p(\varphi,\theta) = 1$, it obeys to the De Mestre's relationship and can be expressed as

$$\beta^{\mathrm{Unif}} = \frac{n_l \mathcal{S}_{\mathrm{t}}}{4} \tag{11}$$

$\mathcal{S}_{\mathrm{t}}$ corresponding to the total area for one leaf.

For a given leaf orientation, the extinction coefficient depends on the angles, Ψ and Θ, under which the leaves are illuminated. The obstruction section $\mathcal{S}_{\mathrm{obs}}$ has to be evaluated according to this particular direction. Relation (9) related to the $\mathbf{e}_x$ direction cannot be used in a straightforward manner. These angles Ψ and Θ are then used for the definition of a new coordinate system $(\mathbf{e}'_{\mathrm{x}}, \mathbf{e}'_{\mathrm{y}}, \mathbf{e}'_{\mathrm{z}})$ as it is represented on Fig. 1(b) with $\mathbf{e}'_{\mathrm{x}}$ corresponding to the incidence direction. New polar and azimuthal angles, φ' and θ', are defined for the direction normal to the leaf.

In viewing the vegetal set under the angles (Ψ, Θ), the mean projected area of one leaf $\mathcal{S}_{\mathrm{obs}}$ (according to $\mathbf{e}'_{\mathrm{x}}$) is estimated in taking into account all possible orientations

$$\mathcal{S}_{\mathrm{obs}} = \frac{S}{4\pi} \int_{\varphi'=0}^{\varphi'=\pi} \int_{\theta'=0}^{\theta'=2\pi} p(\varphi,\theta) |\cos\theta'| |\sin\varphi'| \sin\varphi' \, \mathrm{d}\theta' \, \mathrm{d}\varphi' \tag{12}$$

In this equation $p(\varphi,\theta)$ still rises a problem for the integration. $p(\varphi,\theta)$ is then transformed in $p(\varphi',\theta',\Psi,\Theta)$, using the azimuthal symmetry assumption and the following geometrical relation

$$\cos\varphi = \sin\varphi' \cos\theta' \cos\Psi + \cos\varphi' \sin\Psi \tag{13}$$

Using relations (10), (12) and (13), the extinction coefficient for the different vegetation orientations can be analytically expressed with the view angles which represent the propagation direction for the radiative transfer,

- Planophile:

$$\beta^{\mathrm{Plano}}(\Psi,\Theta) = \beta^{\mathrm{Unif}} \left(\frac{3}{2}\cos^2\Psi + \frac{3}{4}\sin^2\Psi \right) \tag{14}$$

- Erectophile:

$$\beta^{\mathrm{Erect}}(\Psi,\Theta) = \beta^{\mathrm{Unif}} \left(\frac{3}{4}\cos^2\Psi + \frac{9}{8}\sin^2\Psi \right) \tag{15}$$

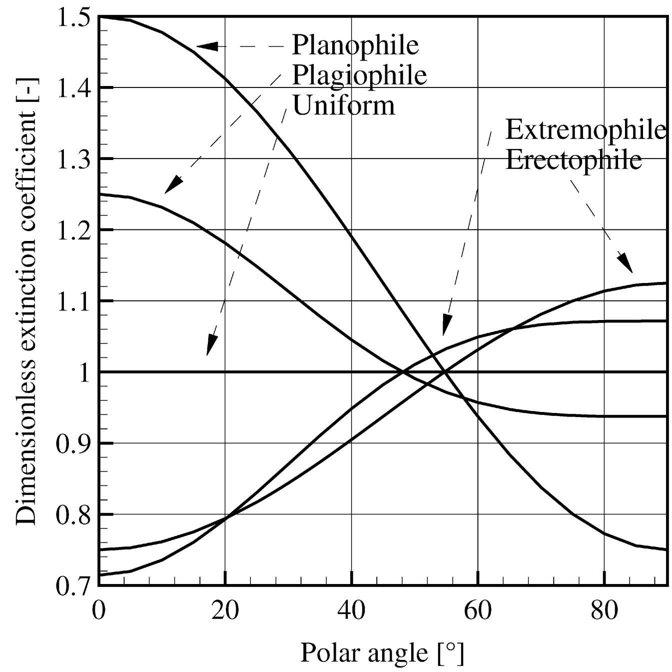


Fig. 3. Dimensionless extinction coefficient according to orientation type.

- Plagiophile:

$$\beta^{\mathrm{Plagio}}(\Psi,\Theta) = \beta^{\mathrm{Unif}} \frac{30}{32} \left(4\cos^2\Psi + 2\sin^2\Psi - \frac{8}{3}\cos^4\Psi - \sin^4\Psi - 4\sin^2\Psi\cos^2\Psi \right) \tag{16}$$

- Extremophile:

$$\beta^{\mathrm{Extre}}(\Psi,\Theta) = \beta^{\mathrm{Unif}} \frac{30}{28} \left(\frac{8}{3}\cos^4\Psi + \sin^4\Psi - 2\cos^2\Psi + 4\sin^2\Psi\cos^2\Psi \right) \tag{17}$$

None of these relations explicitly depends on Θ due to the azimuthal symmetry. The ratio $\beta^{\mathrm{Type}}/\beta^{\mathrm{Unif}}$ is represented on Fig. 3, for the four orientations described by relations (14) to (17) (β^{Type} designates one of the four types considered here). Results show ratios in the range between 0.7 and 1.5. For example, for the planophile case, under a vertical incidence, the extinction coefficient is 50% larger than the one of a turbid media. This difference may have some importance in the radiative transfer involved in forest fires. Moreover, the variations of the extinction coefficient follow the same trends than the probability densities (except for the plagiophile configuration).

Following the works by Biswas [12] a realistic canopy can be represented by a linear combination of the De Wit's functions [7]. Consequently, the extinction coefficient of this new leaf orientation can be expressed by

$$\begin{aligned}\beta^{\mathrm{mixed}}(\Psi,\Theta) = \beta^{\mathrm{Unif}} \bigg[& \left(\frac{15}{4} - \frac{9a}{4} - 3b \right) \cos^2\Psi \\ & + \left(\frac{15}{8} - \frac{9a}{8} - \frac{3b}{4} \right) \sin^2\Psi - \frac{5}{2}(1-a-b)\cos^4\Psi \\ & - \frac{30}{32}(1-a-b)\sin^4\Psi \\ & - \frac{15}{4}(1-a-b)\sin^2\Psi\cos^2\Psi \bigg] \end{aligned} \tag{18}$$

where a and b are real constants defining the new probability density of Eq. (7).

These relationships are useful for predicting the radiation extinction by a leaf set if radiative transfer is treated inside the vegetation canopy by the Finite Volume Method, the Monte Carlo Method or other methods, because they take into account the leaf orientation. However the RTE solution involves huge time costs due to this directional dependency. Therefore, several simplified models suggest to reduce this cost by omitting the directional aspect. In this case, the extinction coefficient is no more expressed as a function of the view angle (Ψ, Θ) but it could be obtained in averaging the above coefficients according to all possible directions as

$$\overline{\beta} = \frac{1}{4\pi} \int\limits_{\Psi=0}^{\Psi=\pi} \int\limits_{\Theta=0}^{\Theta'=2\pi} \beta(\Psi, \Theta) \sin\Psi \,\mathrm{d}\Theta \,\mathrm{d}\Psi \tag{19}$$

Actually, after averaging process the mean extinction coefficient remains the same whatever the leaf orientation, equal to the one of a randomly oriented medium

$$\overline{\beta^{\mathrm{Plano}}} = \overline{\beta^{\mathrm{Erect}}} = \overline{\beta^{\mathrm{Plagio}}} = \overline{\beta^{\mathrm{Extre}}} = \beta^{\mathrm{Unif}} \tag{20}$$

This remark can justify the use of the extinction coefficient for a uniform orientation as a first attempt in simplified models.

3. Numerical method

A second way to derive extinction properties for the vegetation is to compute it numerically. The present numerical tool is based on a 3D ray-tracing method. Numerous rays are followed in the medium, registering the successive paths and the extinction events (only absorption when using the black surface assumption). This technique has been developed in a former study [2] where the properties of a real vegetation medium have been also introduced as input data given by our measurements for absorption, reflection and transmission capacity of the leaves. The same technique is used here in order to identify the effective extinction coefficient of a given medium, but the emphasis is put on leaf orientation or position and only purely absorbing leaves are considered. Two steps are involved:

- the numerical building of the vegetation medium,
- the radiative transfer simulation itself based on the ray-tracing method.

A numerical parallelepipedic domain is considered with leaves distributed inside. The medium between the leaves is transparent, whereas the leaves are participating to the radiative transfer behaving as black radiative surfaces. Then, the De Mestre's relationship [3] can be used with confidence without any correction if a uniform distribution is considered for both spatial position and orientation. Note that the shape of the leaf itself has no actual influence according to our previous study (see [2]), what is important is the total leaf area per unit volume. Since the code has been prepared for the simulation of *Quercus coccifera* at the beginning, the leaf shape is kept typical of this species in the simulation (with a leaf surface of 12.1 cm^2). Rays are initially launched from a random position inside the domain, according to a given incidence direction set by the polar angle. A ray is followed inside the medium until it hits a vegetation element. Once the ray is intercepted by one leaf, absorption occurs. As briefly explained in the introduction, this assumption of a complete absorption is close to the reality, since measurements in the infrared have shown that the mean absorptivity of the vegetation is close to 1 [2]. This has been observed considering six species and spectral measurements between 1000 and 6000 cm^{-1}. For all the studied species, the absorptivity is quite constant, around 0.97, for wavenumbers below 4000 cm^{-1}. Some variations are then observed for higher wavenumbers, but averages based on Planck's means over the infrared range yielded values always higher than 0.90 (between 0.90 and 0.95). These values have been computed using a source like a blackbody up to 1000 K (a temperature close to the one of the flames during vegetation fires even if the true radiation of flame is not the one of a blackbody). This makes the hypothesis of complete absorption in an average sense a correct approximation.

The results obtained for each simulated case are composed of the following data:

- $\mathrm{nb}_{\mathrm{shoot}}$ is the total number of tracked rays,
- $\mathrm{nb}_{\mathrm{out}}$ is the number of rays leaving the medium without being absorbed inside the medium,
- $\mathrm{nb}_{\mathrm{extinct}}$ is the total number of extinction events (only absorption here) that take place inside the leaf set,
- $\mathcal{L}_{\mathrm{Tot}}$ is the total distance traveled by the $\mathrm{nb}_{\mathrm{shoot}}$ rays.

With this ray-tracing method and a great number of followed rays, the effective extinction coefficient β can be estimated for a given medium. A formulation based on the mean free path l defined as the distance traveled for one ray before any absorption event is used:

$$l = \frac{1}{\beta} = \frac{\mathcal{L}_{\mathrm{Tot}}}{\mathrm{nb}_{\mathrm{extinct}}} \tag{21}$$

$\mathrm{nb}_{\mathrm{shoot}}$ and $\mathrm{nb}_{\mathrm{out}}$ are used in order to control the solution. A sensitivity analysis has been carried out prior to the present numerical study, allowing us to verify that the numerical conditions do not have any influence on the results. The following numerical set of parameters has been chosen: 1000 leaves per cubic meters are distributed, the computational domain is a 5 m size cube in which 200,000 rays are followed. The process is repeated on 10 equivalent turbid media and an average is performed on the 10 results. An indication on the variance of the results will be given on one typical case in order to show the uncertainties related to the numerical method.

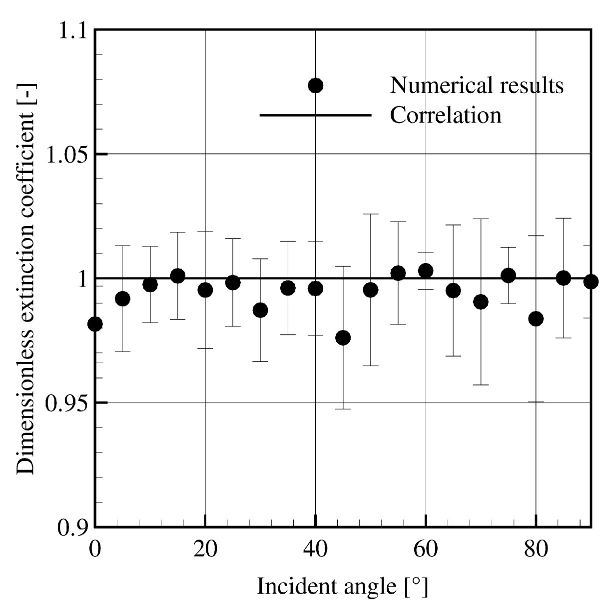


Fig. 4. Dimensionless extinction coefficient for a randomly generated vegetation.

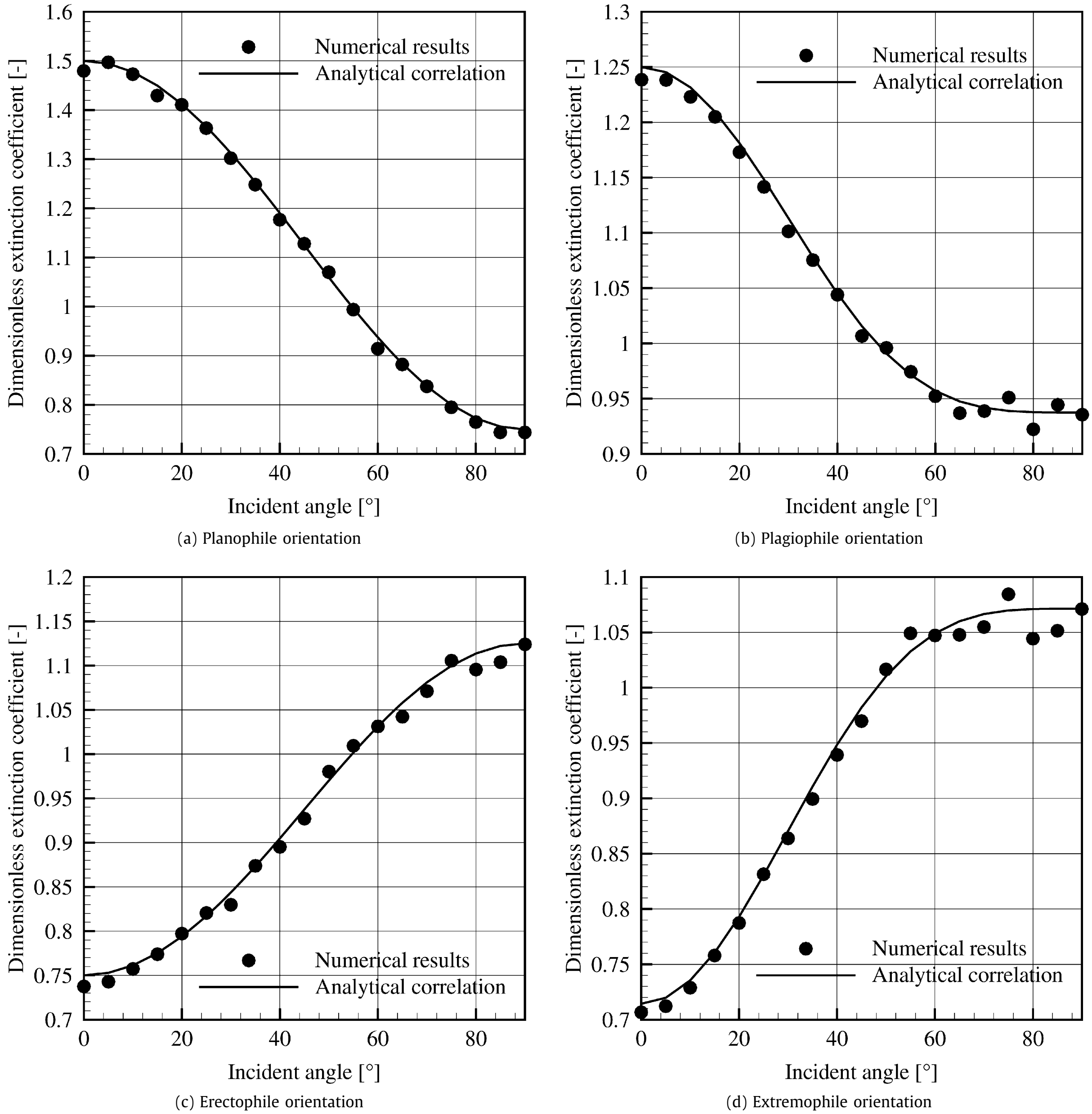


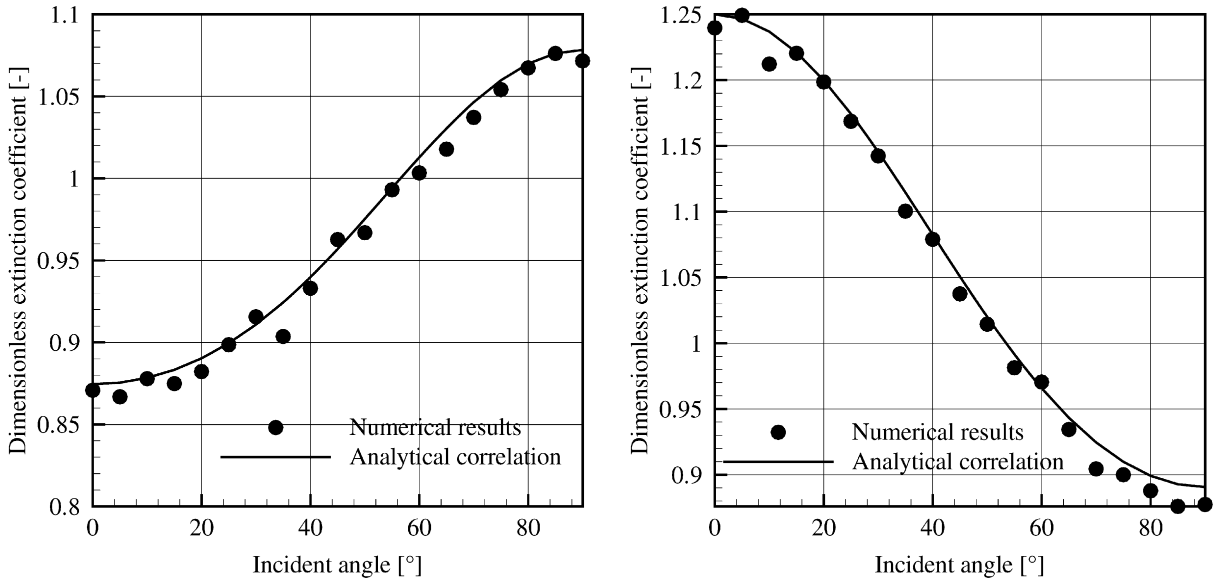
Fig. 5. Dimensionless extinction coefficient, normalized through division with the De Mestre's coefficient, as a function of incidence angle of the incoming radiation, and for various orientations of leaves but with uniform spatial distribution.

4. Numerical results

4.1. Academic situation: fixed orientation and spatial homogeneous distribution

Different computations have been done, they are presented in a dimensionless form as a function of the angle under which the medium is illuminated. This dimensionless extinction coefficient is the ratio between the numerical extinction coefficient and the corresponding value provided by the De Mestre's relationship: $\frac{n_l \mathcal{S}_t}{4}$. The first test concerns a complete random set of leaves (uniform spatial distribution and orientation). Since a random orientation is considered there is no influence of the incidence angle in this validation test and the dimensionless coefficient has to be equal to 1 whatever the direction. As can be seen on Fig. 4, the ratio is close to 1 with statistical uncertainties related to the ray tracing approach ($\pm 2.5\%$) that well bracket the true result. This uncertainty could be decreased, increasing the number of rays, or increasing the number of simulated cases, but this is not really necessary since the actual result is inside the error bars. The same numerical parameters are kept for the ray tracing process when definite orientations are chosen in the followings and similar uncertainties are obtained (no more shown on the figures in order to make the influence of the incidence angle clearer).

Fig. 5 presents the four cases of possible orientation: planophile, plagiophile, erectophile and extremophile. The trends of the curves are similar but extremum values and increase or decrease with the incidence angle depend on the leaf orientation. The maximum extinction is obtained as incidence angle is perpendicular to the main leaf orientation in the planophile and erectophile cases. For these two situations (Figs. 5(a) and 5(b)), a symmetrical trend is observed but the maximal extinction values are not the same due to angular considerations on the unit sphere where angles are defined (consider a given polar angle and azimuth ranging from 0



(a) $a = 0$, $b = 0.75$, $c = 0.25$

(b) $a = 0.5$, $b = 0.25$, $c = 0.25$

Fig. 6. Dimensionless extinction coefficient for two cases of combined orientations with orientation function derived according to the formalism by Biswas [12].

to 360°, the resulting circle on the unit sphere is not the same near the Z axis ($\theta = 0°$) and perpendicular to the Z axis ($\theta = 90°$) and consequently incidence consideration is not symmetrical for $\theta = 0°$ on planophile leaves and for $\theta = 90°$ on erectophile leaves). The role of leaf orientation is obvious since relative discrepancy from the random orientation (dimensionless coefficient equal to one) reaches values up to −30% and +50%, with maximum shift observed for the planophile orientation.

Comparisons between analytical formulation (continuous line) given by Eqs. (14) to (17) and ray tracing results (black spots) show a good agreement, validating our numerical identification of the extinction coefficient in various situations. Spots only shift from the theoretical values in a range lying in the uncertainty limits previously seen (Fig. 4).

A supplementary validation is presented on Fig. 6 where two examples of mixings of different orientations is used. The cases $a = 0$, $b = 0.75$, $c = 0.25$ and $a = 0.5$, $b = 0.25$, $c = 0.25$ have been chosen for the illustration, using the formalism by Biswas [12]. Comparison between numerical prediction and analytical relationship (18) shows a good agreement again. Variations are of course governed by the mixing type, and the most representative species. The assumption of a linear combination of the orientation function gives results well reproduced by the numerical method. This also demonstrates that the numerical tool is ready to be used on any leaf orientation, a required validation before the prediction of extinction properties on true trees in a next step.

4.2. Simulation of non-uniform distributions and orientations

The previous academic situation is now extended to a heterogeneous leaf distribution in space. A set of leaves simulating a crown is given in a box, and the effective extinction coefficient will be identified. The actual distribution will not be the one of a true tree, but some heterogeneities will be introduced according to vertical and radial positions in the box. In an actual tree, one may suppose that leaves are mainly distributed on the periphery of the crown. This is for example suggested by Zeide in [13], who explains that the peripheral branches do not spread to provide light to the interior. As a consequence foliage tends to occupy the outer space while creating a leafless core inside. The present representation uses this suggestion, in a simplified approach, in order to test the effect of heterogeneity in the spatial distribution of leaves. We consider a sphere representing the crown, where leaves may be uniformly distributed or confined in the external part of the crown, in a region with a given thickness hold between two concentric spheres. The inner sphere is characterized by its radius $R_{\min}$ normalized by the radius of the outer sphere. The internal part of the tree is free of leaves. Actually, computations have been carried out with the radius of the inner sphere varying from 0 (leaves are then uniformly distributed, the distribution is in fact still homogeneous) to 1 (all leaves are artificially located on the periphery of the crown with an empty core). The next step would be to introduce LAI (Leaf Area Index) as a function of radial and vertical position in order to take into account a more realistic representation. Another possible improvement would take into account branches, through the introduction of cylindrical shapes. These steps are planned in a next work. The present simplified distribution is only used in order to generate some spatial heterogeneity in the distribution and to study its influence. The box size of the computational domain is determined by the "tree" size itself. It is still a cubic domain, but this time with a size defined in order to hold the tree: a cube with a 5 m size in the present case. The interest of this approach is that a complete forest domain could be built in a further work, by successive definitions of neighboring boxes and the corresponding spatial evolution of the extinction coefficient could be finely introduced in a model propagation on a large scale.

Figs. 7 and 8 present the results obtained for leaves with a random and a planophile orientation respectively. The influence of $R_{\min}$, and consequently of the heterogeneous distribution, is obvious. This is especially clear on Fig. 7, since in this case leaves are randomly oriented and heterogeneity in the distribution is the only studied parameter. Extinction coefficient is still plotted as a function of the incidence angle and normalized using the De Mestre's reference value. The constant line is of course the result of the De Mestre's relationship on Fig. 7 (no variation with the incidence angle). The relationship for a planophile orientation is also plotted on Fig. 8 (based on Eq. (14)). On Fig. 7 deviation from the De Mestre's relationship reaches values between −17 and +12%, depending on the incidence angle of radiation. The discrepancy is even higher with the planophile orientation: from −35 to +50%.

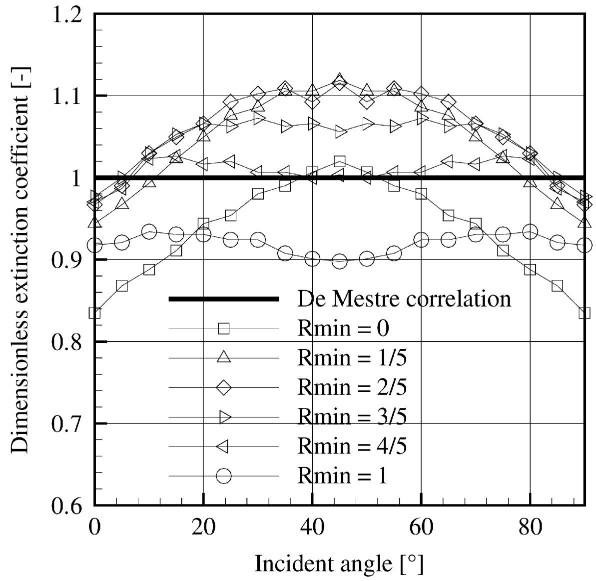


Fig. 7. Dimensionless extinction coefficient for a set of leaves with random orientation and heterogeneous spatial distribution. Leaves only distributed on the peripheral part of the crown, between two concentric spheres, the inner sphere being defined by its normalized radius $R_{\min}$. Comparison with a uniform distribution ($R_{\min} = 0$) and the De Mestre's relationship.

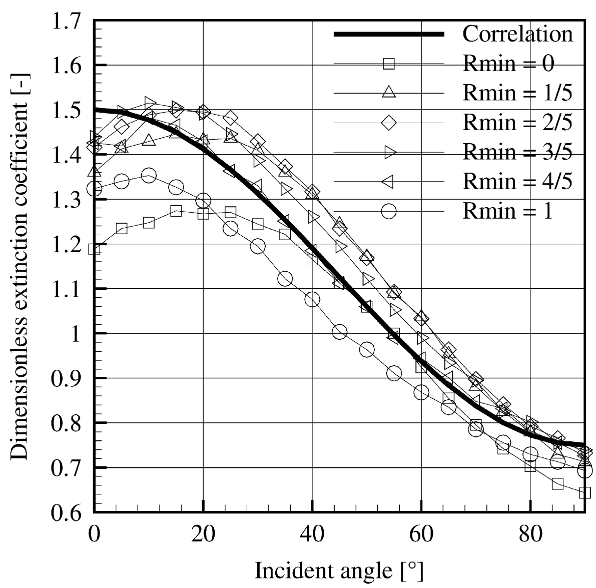


Fig. 8. Same caption as Fig. 7 but for leaves with a planophile orientation.

One can see that the numerical results computed with $R_{\min} = 0$ are not following the curves given by the analytical relationships. It is not contradictory to the above analysis related to the validation of the numerical method. This observation can be explained by the fact that leaves are supposed to be distributed in a spherical domain whereas the computational domain is a cubic box including the sphere, so that the distribution is homogeneous in the "tree" but not in the whole domain. This is in fact an other illustration of the influence of a heterogeneous distribution in space. This is not a problem for further extension of the method: each box will contain a tree (or a set of species) without overlapping of the cubic boxes and the shape of the domains will have no influence on the final definition of the extinction coefficient field.

Concerning the influence of $R_{\min}$ on Fig. 7, the trend is not linear since the extinction coefficient first increases with $R_{\min}$ (as leaves are distributed more and more on the outer part of the crown), then it decreases for values of $R_{\min}$ above a threshold between 2/5 and 3/5. This remark could be associated to some observations in the domain of ecology according to which the vegetation will distribute its leaves as to optimize its ability to capture the light. Leaves only on the peripheral area will not succeed in absorbing all the available light, but on the contrary leaves located near the core would not receive light due to shadow effects. The same phenomenon is observed here. Of course for a further application in the frame of fire propagation due to radiation, this optimization regarding light absorption will become a drawback for limiting the infrared absorption. Finally, the role of the heterogeneity in the distribution is observed but the extinction coefficient does not vary much for $R_{\min}$ between 1/5 and 3/5.

For the planophile case (Fig. 8) the angular influence is of course different because of the role of the leaf orientation. The deviation is larger when compared to the De Mestre's relationship, but the discrepancy between the different cases (from $R_{\min} = 0$ to 1) is less important. Here again there is range for $R_{\min}$ between 1/5 and 3/5 where the radiation extinction is the highest for the same reasons as above explained.

5. Conclusion

The effective extinction coefficient of a vegetation medium has been studied. Analytical relationships have been derived for various leaf orientations, extending the well-known De Mestre's formulation for random leaf distributions to planophile, plagiophile, erectophile, extremophile cases and mixing of these orientations. The incidence angle is now taken into account and its effective influence has been studied, indicating deviations up to 50% from the reference random case. Such an influence should be taken into account in the properties used as input in the propagation models.

A numerical tool based on a ray tracing method has been also developed and validated for these orientations when the spatial distribution of leaves is homogeneous. When heterogeneities are introduced in the position of the leaves in a tree, this numerical method still allows the identification of an effective extinction coefficient, whereas analytical formulations are no more possible. Considering leaves in a crown on the periphery of a tree, we have observed deviation again up to 50% from the analytical formulae yielded for the different orientations.

This numerical approach is now ready for an application on a real tree, or set of trees, defined on the basis of true morphological properties: accurate orientation function and LAI data. Fine spectral variations are also available and could provide further refinement possibilities.

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